HW 10: NUMBER THEORY

1. Find all pairs of real numbers (a, b) such that $a[bn] = b[an]$ for all positive integers n.

2. Prove that the equation $x^2 = y^3 + 7$ has no integer solutions.

3. Prove that for any positive integer n other than 2 or 6,
$$\Phi(n) \geq \sqrt{n}.$$

4. Prove that for every n, there exist n consecutive integers each of which is divisible by two different primes.

5. An equilateral triangle of side length n is drawn with sides along a triangular grid of side length 1. What is the maximum number of grid segments on or inside the triangle that can be marked so that no three marked segments form a triangle?

6. A coin is tossed n times. What is the probability that two heads will turn up in succession somewhere in the sequence?